Measurement of the mass of the τ-lepton and an upper limit on the mass difference between τ^+ and τ^-

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L. Y. Dong,[11] R. Dowd,[23] J. Dragic,[9] A. Drutskoy,[5] S. Eidelman,[2] Y. Enari,[24] D. Epifanov,[2]
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H. Guler,[8] R. Guo,[27] J. Haba,[9] K. Hara,[9] T. Hara,[35] Y. Hasegawa,[43] N. C. Hastings,[50]
K. Hasuko,[39] K. Hayasaka,[24] H. Hayashii,[25] M. Hazumi,[9] T. Higuchi,[9] L. Hinz,[20] T. Hojo,[35]
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1

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Abstract

The mass of the τ-lepton has been measured in the decay modes $\tau \to 3\pi\nu_\tau$ and $\tau \to 3\pi\pi^0\nu_\tau$ using a pseudomass technique. The preliminary result is $1776.71 \pm 0.25(\text{stat}) \pm 0.62(\text{syst})$ MeV. The preliminary value of an upper limit on the relative mass difference between positive and negative τ leptons is $|(M_{\tau^+} - M_{\tau^-})|/M_{\text{average}}$ is 5.0×10^{-4} at 90% CL.

PACS numbers:

INTRODUCTION

Masses of quarks and leptons are fundamental parameters of the Standard Model (SM). In the SM high precision measurements of the mass, lifetime and leptonic branching fractions of the τ lepton can be used to test lepton universality. The present PDG value of the τ mass [1] is dominated by the result of the BES Collaboration [2] and has an accuracy about 0.3 MeV. The high statistics of the Belle data allow a measurement with the same level of accuracy. The methods used for the τ mass measurement are different for the BES and the Belle experiments: BES analysed the cross section for τ pair production near threshold while in Belle the four-momenta of the visible τ decay products at $\sqrt{s}$=10.58 GeV are used. This leads to different sources of systematic uncertainties. Eventually, by combining these two measurements we could significantly improve the accuracy of the τ mass deterination.

The analysis of individual τ lepton decays allows to measure the masses of positive and negative τ's separately and test the CPT theorem. A similar test was performed by OPAL at LEP [3] with the result $(M_{\tau^+} - M_{\tau^-})/M_{\text{average}} < 3.0 \times 10^{-3}$ at 90% CL. The high statistics and quality of the Belle data allow us to improve this limit significantly.

To determine the τ mass we use a pseudomass technique that was first employed by the ARGUS [4] collaboration. This technique relies on the reconstruction of the invariant mass and energy of the hadronic system in hadronic τ decays.

THE PSEUDOMASS METHOD

In a hadronic τ decay (see Fig. 1) the τ lepton mass M_τ is related to the four-momentum of the resulting hadronic system X by the formula

$$M_\tau^2 = M_X^2 + M_\nu^2 + 2E_X E_\nu - 2P_X P_\nu \cos\theta \tag{1}$$

where M_X, E_X and P_X are the invariant mass, energy and absolute value of the momentum of the hadronic system respectively; M_ν, E_ν and P_ν are the same quantities for the neutrino and θ is the angle between the momenta of the neutrino and hadronic system.
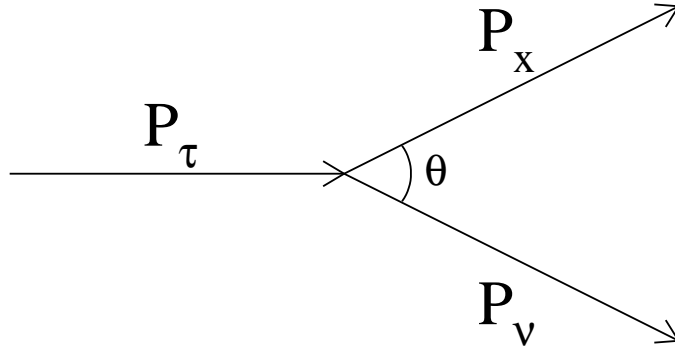

FIG. 1: Illustration for the variable definitions used in the Eq.(1)

If $M_\nu = 0$ we have $P_\nu = E_\nu = E_\tau - E_X$. This gives the following expression for the τ mass:

$$M_\tau^2 = M_X^2 + 2(E_\tau - E_X)(E_X - P_X \cos\theta). \tag{2}$$

The τ lepton energy E_τ is obtained from the energy of the electron or positron beam, E_{beam}, in the center-of-mass (c.m.) frame. All other kinematic variables listed above will also be evaluated in the c.m. frame of the colliding beams.

If we set the unknown value of $\cos\theta$ in equation (2) equal to 1, the right side of (2) will be smaller than the true value for M_τ^2. Therefore, the estimator of the τ mass (the so-called pseudomass) used in the analysis
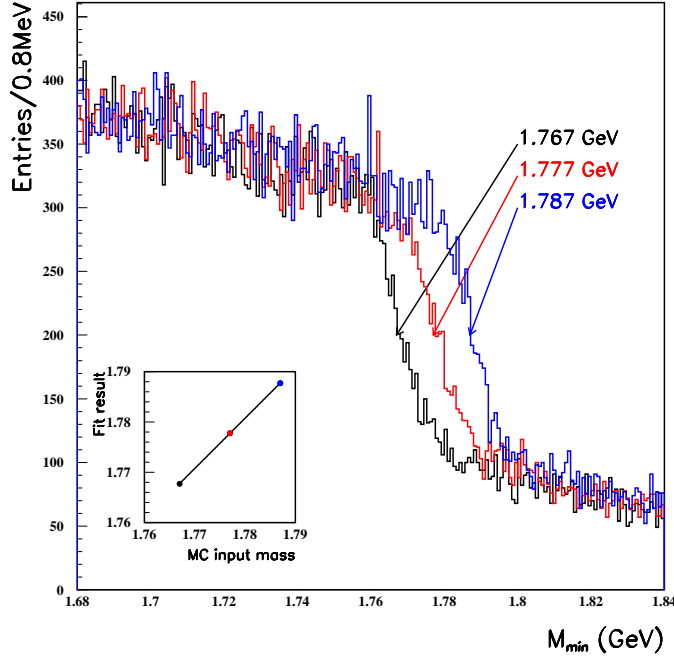
FIG. 2: The Monte Carlo distributions of the pseudomass M_{min} for the $\tau \to 3\pi^\pm\nu$ decays and the input τ masses equal to 1.767, 1.777 and 1.787 GeV, respectively. The fitted values of the τ mass versus the input mass used in the MC simulation are shown together with a straight line fit.

$$M_{\mathrm{min}} = \sqrt{M_X^2 + 2(E_{\mathrm{beam}} - E_X)(E_X - P_X)} \tag{3}$$

should be less than or equal to the τ lepton mass. In the absence of initial and final state radiation and assuming a perfect measurement of the four-momentum of the hadronic system, M_{min} is distributed below the τ mass and has an edge at M_τ. The initial and final state radiation as well as the finite momentum resolution of the detector smear the shape of the edge for M_{min} around M_τ. The contributions from background processes in the selected $\tau^+\tau^-$ sample have smooth behavior near the τ mass. We can use the threshold position obtained from the fit to the experimental M_{min} distribution as an estimator of the τ mass.

To illustrate this method in the Belle environment we performed simulations of τ decays into three charged pions and a neutrino with three different input τ masses: the nominal PDG value 1.777 GeV, 1.767 GeV and 1.787 GeV[9]. The generated events were passed through the full Belle detector response simulation and reconstruction procedures. The resulting M_{min} distributions for the three input τ masses given above are shown in Fig. 2. The inset in Fig. 2 shows the dependence of the fitted masses obtained from the M_{min} distributions on the input masses used in the MC simulation. The result of the fit of this

dependence to the linear function $f = a_0 + a_1 \times x$ gives $a_0 = (0.1 \pm 0.2) \times 10^{-2}$ GeV and $a_1 = 1.000 \pm 0.001$.

After obtaining the value of the threshold position from the fit to the experimental $M_{\min}$ distribution we correct it by the value, obtained from the Monte Carlo, which is equal to the difference between the true input value of the τ mass used in the MC simulation and the threshold position obtained from the simulated data.

EXPERIMENTAL PROCEDURE

Analysis presented here is based on the data taken with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. The total integrated luminosity used in the analysis is 253 fb^{-1}.

A detailed description of the Belle detector is given elsewhere [5]. We mention here only the detector components essential for the present analysis.

Charged tracks are reconstructed from hit information in a central drift chamber (CDC) located in a 1.5 T solenoidal magnetic field. The z axis of the detector and the solenoid are along the positron beam direction, with positrons moving in the $-z$ direction. The CDC measures the longitudinal and transverse momentum components (along the z axis and in the $r\phi$ plane, respectively). Track trajectory coordinates near the collision point are provided by a silicon vertex detector (SVD). Photon detection and energy measurements are performed with a CsI(Tl) electromagnetic calorimeter (ECL). Identification of kaons is based on the information from the time-of-flight counters (TOF) and silica aerogel Cherenkov counters (ACC). The ACC provides good separation between kaons and pions or muons at momenta above 1.2 GeV. The TOF system consists of a barrel of 128 plastic scintillation counters, and is effective in K/π separation mainly for tracks with momentum below 1.2 GeV. The lower energy kaons are also identified using specific ionization (dE/dx) measurements in the CDC. Identification of electrons is made using combined information from ECL, ACC, TOF and CDC [6]. The magnet return yoke is instrumented to form the K_L and muon detector (KLM), which detects muon tracks [7] and provides trigger signals.

The signal events were efficiently triggered by several kinds of track triggers that require two or more CDC tracks with combinations of TOF hits, ECL clusters or its energy sum. Here, we do not eliminate any events using the trigger condition information. The trigger conditions are complementary to each other for the detection of four-prong events in the present case. We can expect a high trigger efficiency ($\sim 95\%$) by combining them.

We used only on-resonance data because the absolute beam energy calibration is known for this data sample better than for the off-resonance data taken 60 MeV below the $\Upsilon(4S)$ ($\sqrt{s} = 10.58$ GeV).

$\tau^+\tau^-$ events were selected where one τ lepton decays leptonically into $l\bar{\nu}_l\nu_\tau$. The other τ lepton decays into hadronic decay modes with 3 charged pions and a neutrino or 3 charged pions with 1 neutral pion and a neutrino.

The preselection of events is based on the following criteria:

(a) Visible reconstructed energy $E_{sum} > 0.18\sqrt{s}$;

(b) Number of well-reconstructed charged tracks greater than 2;

(c) The sum of the z components of each good track and good photon momenta is required to satisfy $|P_z| < 0.5\sqrt{s}$.

Conditions (a) and (c) are calculated in the c.m. frame.
The criteria for good charged tracks are:

- $p_T > 100$ MeV;

- Impact parameters $\Delta r < 2$ cm, $|\Delta z| < 4$ cm.

Good photons are defined as ECL clusters with energy greater than 100 MeV that are not associated with charged tracks. The angular acceptance for photons is $17° < \theta < 150°$.
The preselection cuts suppress Bhabha, $\mu^+\mu^-$ and two-photon events.
After the preselection the following cuts were applied.

- Total charge equal to zero;

- Number of leptons (muons or electrons) equal to one;

- Number of charged pions equal to three;

- Number of charged kaons and protons equal to zero;

- Number of K_S's equal to zero;

- Number of π^0's equal to zero for the $\tau \to 3\pi\nu_\tau$ decay mode and equal to one for the $\tau \to 3\pi\pi^0\nu_\tau$ mode.
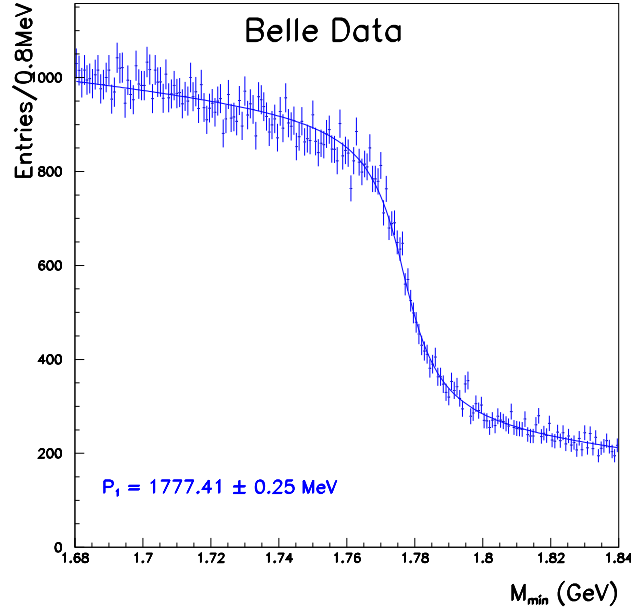
FIG. 3: The pseudomass distribution $M_{\min}$ for the $\tau \to 3\pi^\pm\nu$ decays. The points with error bars are data and the solid line is the result of the fit with function (4).

The $M_{\min}$ distribution for the $\tau \to 3\pi\nu$ data is shown in Fig. 3. A fit was performed to the data with the function
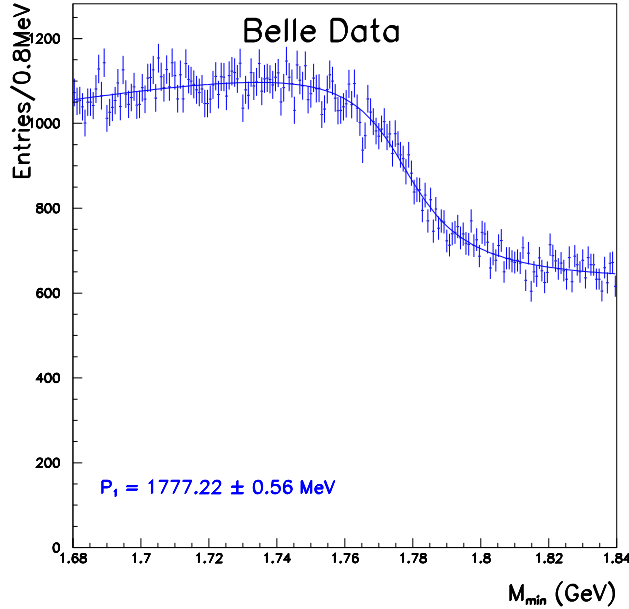
FIG. 4: The pseudomass distribution $M_{\min}$ for the $\tau \to 3\pi^{\pm}\pi^0\nu$ decays. The points with error bars are data and the solid line is the result of the fit with function (4).

$$F(x) = (p_3 + p_4 \times x) \times \arctan((x - p_1)/p_2) + p_5 + p_6 \times x \qquad (4)$$

The value of the parameter p_1 obtained from the fit is $p_1 = 1777.41 \pm 0.25$ MeV.

The difference between the threshold position obtained from using equation (4) and the true value of the τ mass obtained from MC is equal to $\delta p_1 = 0.70\pm0.40$ MeV. The uncertainty in δp_1 is dominantly due to limited Monte Carlo statistics (which is about $1/2$ of the data) and the systematics of the fit procedure (choice of the fit range, shape of the threshold and background function).

The distribution of $M_{\min}$ for the $3\pi\pi^0\nu_\tau$ decay mode is shown in Fig. 4 together with the results of the fit with the same function.

The value of the τ mass estimator for this decay mode is $p_1 = 1777.22\pm0.56$ MeV which is consistent within errors with the result from the $\tau \to 3\pi^{\pm}\nu$ decay mode. As the statistical error for the $3\pi^{\pm}\pi^0\nu$ mode is significantly larger than for the $3\pi^{\pm}\nu$ one, we will concentrate on the former decay mode only.

SYSTEMATIC UNCERTAINTIES

The following contributions to the overall systematic uncertainty were considered:

- Calibration of the tracking system. We used muons from the decay of $\Upsilon(1S)$, which are decay products of $\Upsilon(2S, 3S)$ to $\Upsilon(1S)\pi^+\pi^-$. The peak position of $\Upsilon(1S)$ is shifted from the nominal PDG value by -4.5 ± 2.3 MeV.

  The invariant mass distribution of $\mu^+\mu^-$ for the $\Upsilon(1S)$ peak is shown in Fig. 5. As the $\Upsilon(1S)$ is produced almost at rest we use the relative mass shift of the visible peak position from the PDG value as an estimate of the accuracy of the tracking
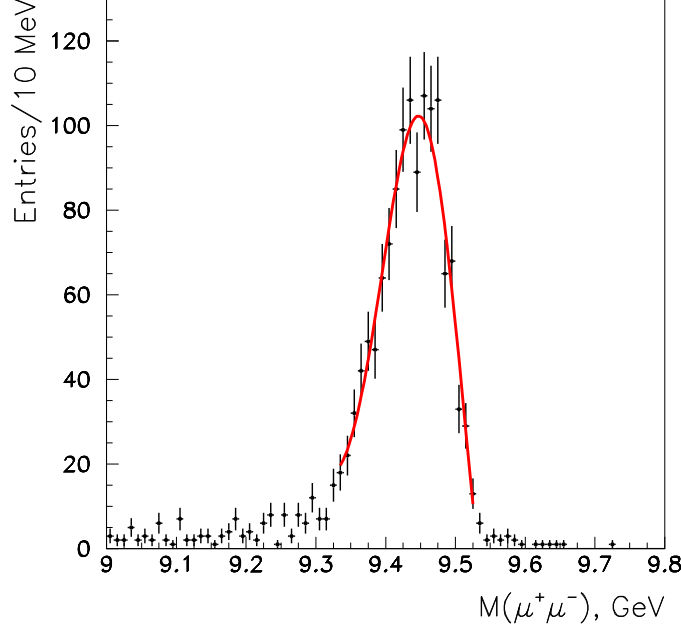
FIG. 5: The invariant mass distribution for $\Upsilon(1S) \to \mu^+\mu^-$

calibration, i.e. $\Delta M_\Upsilon / M_\Upsilon = \Delta P_\mu / P_\mu$. The next assumption is that the relative shift of the momentum of the three pion system is equal to the relative shift of the single track momentum, i.e. $\Delta P_{3\pi} / P_{3\pi} = \Delta P_\mu / P_\mu$. This allows us to propagate the shift of $P_{3\pi}$ to a shift of $M_{\min}$. In order to estimate the shift in the threshold position we finally take $P_{3\pi} \approx P_\tau$ and $M_{\min} \approx M_\tau$, because this is the region that is sensitive to the τ mass. According to the above arguments, the relative shift of the $\Upsilon(1S)$ mass, which is equal to 4.5/9460.3 gives a systematic error from tracking of 0.39 MeV.

- Choice of the fit range and the shape of the threshold function for the τ mass estimation. In addition to the function $\arctan((x - p_1)/p_2)$, where p_1 and p_2 stand for the mass position and resolution respectively, we also tried the parametrizations $(x - p_1)/\sqrt{p_2 + (x - p_1)^2}$ and $1/(1 + exp((x - p_1)/p_2))$. To esimate the value of the correction to our estimator of the τ mass and its uncertainty, we used a Monte Carlo $\tau^+\tau^-$ sample with a statistics of approximately one half compared to the data sample and an input τ mass of 1777.0 MeV. We performed fits to the data and MC samples using three different fit functions mentioned above in five ranges of $M_{\min}$.

  From the fit to the MC distribution by Eq. (4) we get a difference between the visible threshold position and the true input τ mass of $\delta p_1 = 0.70$ MeV. The variation of the fit ranges and threshold function gives a variation of the δp_1 within a $\pm$ 0.40 MeV range.

  We correct the estimator value p_1 obtained from the fit by equation (4) to the data by 0.70 MeV and take the value of 0.40 MeV as an estimate of the systematic uncertainty.

- Uncertainty in the beam energy.

  For the estimation of this uncertainty we used the internal Belle analyses of the full reconstructed B decays for the energy calibration. In these analyses the reconstructed B meson energies were compared with the beam energies supplied by KEKB. The

9

conclusion from these analyses is that the beam energy is known with accuracy better than 1.5 MeV. This uncertainty can be translated to an uncertainty in the τ mass from the following formula

$$\sigma(M_{\min}) = \frac{E_X - P_X}{M_{\min}}\sigma(E_{\text{beam}}). \tag{5}$$

Near the threshold in the $M_{\min}$ distribution we can set $E_X \approx E_{\text{beam}}$, $M_X \approx M_\tau$ and $M_{\min} \approx M_\tau$ which gives $\sigma(M) \approx 0.17\sigma(E_{\text{beam}})$. For $\sigma(E_{\text{beam}}) = 1.5$ MeV we find $\sigma(M_\tau) \approx 0.26$ MeV.

To check the relation $\sigma(M) \approx 0.17\sigma(E_{\text{beam}})$, we performed a simulation of τ decays with different E_{beam} values. We repeated the fit procedure for all MC samples assuming $E_{\text{beam}} = M(\Upsilon(4S))/2$ and plotted the fit results versus the beam energy used in the simulations in Fig. 6. The result of the straight line fit to the plot in Fig. 6 gives the value of the slope of $P_2 = 0.1753 \pm 0.0002$, which is consistent with the analytical calculation.
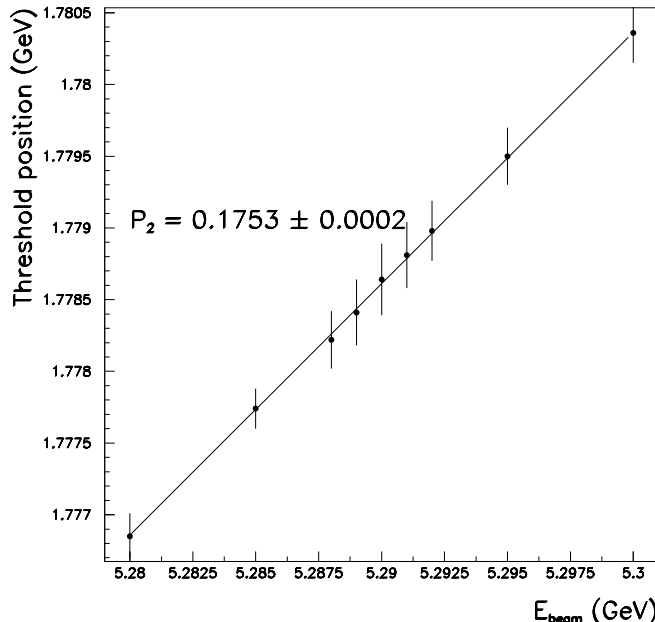
FIG. 6: The dependence of the fit value of the threshold position on the beam energy used for the event simulation. The line is a result of a straight line fit to this dependence.

- Systematic uncertainties coming from misidentified τ decays are negligible, since their $M_{\min}$ distributions show no significant structure in the region of the τ mass. In Fig. 7 the MC $M_{\min}$ distributions are shown for correctly identified pions and incorrectly identified particles. These distributions are obtained from the same MC sample. The background from non-$\tau^+\tau^-$ events can also be neglected.

Adding all these uncertainties in quadrature results in a total systematic error of 0.62 MeV. The final result is $M_\tau = 1776.71 \pm 0.25(\text{stat}) \pm 0.62(\text{syst})\text{MeV}$.
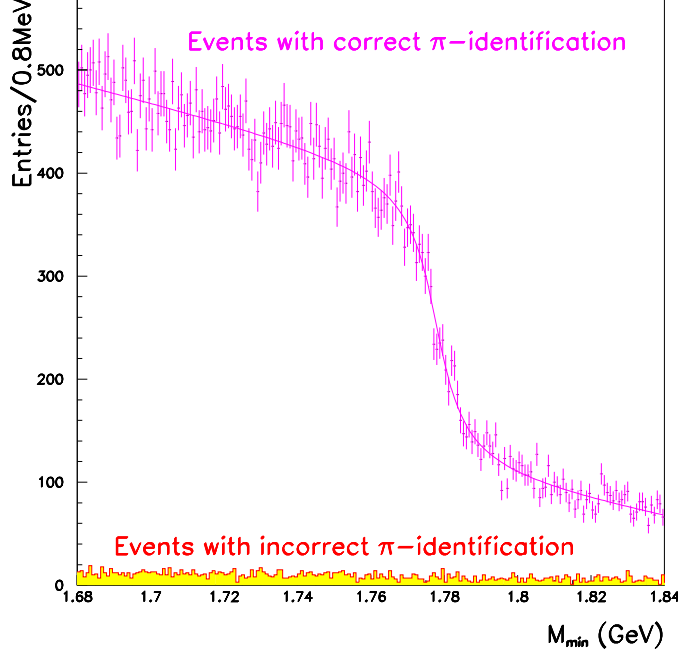
FIG. 7: The MC distribution of the pseudomass $M_{\min}$ for the decays $\tau \to 3\pi^{\pm}\nu$ with correctly identified pions. The filled histogram shows the contribution from incorrectly identified particles.

CPT TEST

The pseudomass method allows a separate measurement of the masses of the positively and negatively charged τ leptons.

A mass difference between positive and negative τ leptons would result in a difference in the energy between the τ's produced in the e^+e^- collision. This in principle makes the assumption $E_{\tau} = E_{\text{beam}}$ invalid. The distributions of the $M_{\min}$ for positive and negative τ's decaying into $3\pi\nu$ are shown in Fig. 8 together with the results of the fit.

Good agreement between the distributions for τ^+ and τ^- is seen. The mass difference obtained from the independent fits to these distributions is $M_{\tau^+} - M_{\tau^-} = -0.12 \pm 0.45$ MeV.

Most sources of systematic errors affect the result for positive and negative τ leptons in the same way, so that their contributions to the mass difference cancel. However, particles and antiparticles interact differently with the detector material.

To estimate a systematic shift in the measurement of particle and antiparticle momenta we compared the peak positions of $D^0 \to K^-\pi^+$ and $\bar{D}^0 \to K^+\pi^-$, $\Lambda_c \to pK^-\pi^+$ and $\bar{\Lambda}_c \to \bar{p}K^+\pi^-$, $D^+ \to \phi(1020)\pi^+$ and $D^- \to \phi(1020)\pi^-$, $D_S \to \phi(1020)\pi^+$ and $\bar{D}_S \to \phi(1020)\pi^-$.

The average relative mass shift from the decay modes listed above is about 0.8×10^{-4}, which gives a systematic uncertainty in the mass difference between τ^+ and τ^- of 0.15 MeV.

Adding the statistical and systematic errors in quadrature we obtain $M_{\tau^+} - M_{\tau^-} = -0.12 \pm 0.47$ MeV.

This result can be expressed as an upper limit on the relative mass difference [8]

$$|(M_{\tau^+} - M_{\tau^-})|/M_{\text{average}} < 5.0 \times 10^{-4} \quad \text{at 90\% CL.} \tag{6}$$

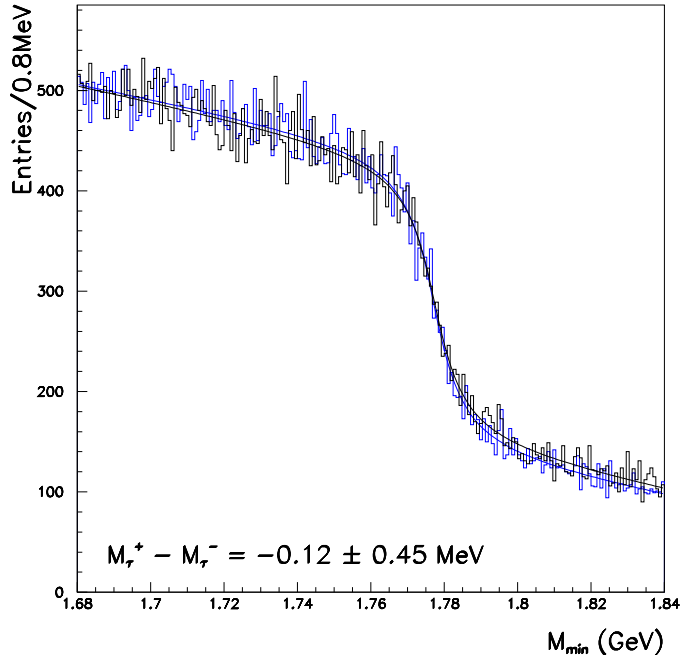Without assuming CPT invariance it is no longer obvious that the charges and masses of

11

FIG. 8: The distributions of the pseudomass $M_{\min}$ for the decays $\tau \to 3\pi^{\pm}\nu$ separately for positive and negative τ decays. In blue the distribution for positive, in black for negative τ decays is shown. The solid lines are the results of the fit with function (4).

positive and negative τ decay products should be the same. Good agreement of the $M_{\min}$ distributions for positive and negative $\tau \to 3\pi\nu$ decays shows that at the present level of experimental accuracy CPT invariance is respected.

RESULTS

We have measured the mass of the τ lepton from the pseudomass distributions of τ decays into three charged pions and neutrino. The result is

$$M_\tau = 1776.71 \pm 0.25(\text{stat}) \pm 0.62(\text{syst}) \text{ MeV}. \tag{7}$$

We obtained an independent measurement of the positive and negative τ mass. The measured values are consistent and an upper limit on the relative mass difference is $|(M_{\tau^+} - M_{\tau^-})|/M_{\text{average}}$ is 5.0×10^{-4} at 90% CL.

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